CBCS SCHEME

USN ☐☐☐☐☐☐☐☐☐☐

22MCA11

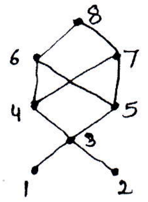
First Semester MCA Degree Examination, Dec.2023/Jan.2024

Mathematical Foundation for Computer Applications

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	**M**	**L**	**C**
Q.1	**a.**	Define a set, an empty set and a singleton set, with an example for each.	**6**	**L3**	**CO1**
	b.	For any two sets A and B, prove that (i) $\overline{A} \cap \overline{B} = \overline{A \cup B}$. (ii) $\overline{A} \cup \overline{B} = \overline{A \cap B}$.	**7**	**L3**	**CO1**
	c.	Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$.	**7**	**L3**	**CO1**
		OR			
Q.2	**a.**	Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 4, 6, 8\}$, $B = \{2,4,5,9\}$. Compute the following : (i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$ (iv) $\overline{A}$	**6**	**L3**	**CO1**
	b.	A total of 1232 students have taken a course in Java, 879 in C and 114 in C++. Further, 103 have taken courses in both Java and C, 23 in both Java and C++ and 14 in both C and C++. If 2092 students have taken at least one of Java, C and C++, how many students have taken a course in all three subjects.	**7**	**L3**	**CO1**
	c.	State pigeon-hole principle. Show that if 50 books in a library contain a total of 27551 pages, one of the books must have atleast 552 pages.	**7**	**L3**	**CO1**
		Module – 2			
Q.3	**a.**	Write the converse, inverse and contrapositive of, "If 2 is an even number then 7 is a prime number".	**6**	**L2**	**CO3**
	b.	Define tautology. Show that $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ is a tautology.	**7**	**L2**	**CO3**
	c.	If $A = \{1, 2, 3, 4, 5\}$ is the universe of disclosure, determine the truth values of, (i) $\forall x \in A, (x + 2 < 10)$ (ii) $\exists x \in A, (x + 2 = 10)$ (iii) $\forall x \in A, (x^2 \leq 25)$	**7**	**L2**	**CO3**

		OR			
Q.4	**a.**	Show that for any three propositions p, q, r, $p \to (q \wedge r) \Leftrightarrow [(p \to q) \wedge (p \to r)]$.	6	L2	CO3
	b.	Negate and simplify : (i) $\forall x, [p(x) \wedge \neg q(x)]$. (ii) $\exists x, [\{p(x) \vee q(x)\} \to r(x)]$	7	L2	CO3
	c.	Show that the following argument is valid : "If today is Tuesday, I have a test in Mathematics or Economics. If my Economics professor is sick, I will not have a test in Economics. Today is Tuesday and my Economics professor is sick. Therefore, I have a test in Mathematics".	7	L2	CO3
		Module – 3			
Q.5	**a.**	Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by 'aRb iff a is a multiple of b'. Write down R as a set of ordered pairs. Write matrix and digraph of R.	6	L3	CO1
	b.	Consider $A = \{1, 2, 3, ...11, 12\}$. The relation R on A is defined as $(x, y) \in R$ iff x – y is a multiple of 5. Verify that R is an equivalence relation.	7	L3	CO1
	c.	Let $A = \{2, 3, 4, 6, 8, 12, 24\}$ and R denote the partial order of divisibility 'xRy iff x divides y'. Find R and draw its Harse diagram.	7	L3	CO1
		OR			
Q.6	**a.**	Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. Relations R and S from A to B are given by, $M_R = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$, $M_S = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$ Find (i) $R \cup S$ (ii) $R \cap S$ (iii) $\bar{S}$	6	L3	CO1
	b.	Let $A = \{1, 2, 3, 4\}$ & $R = \{(1,1), (1,2), (2,3), (3,4)\}$, $S = \{(3,1), (4,4), (2,4), (1,4)\}$ be relations on A. Determine the relations $R \circ S$, $S \circ R$, R^2, S^2 and draw their diagraphs.	7	L3	CO1
	c.	Consider the POSET whose Harse diagram Fig. Q6 (c) is as given below. Consider $B = \{3, 4, 5\}$. Find (i) Maximal elements (ii) Minimal elements (iii) Greatest elements (iv) Least elements (v) Upper bound of B (vi) Lower Bounds of B (if they exist)  Fig. Q6 (c) .	7	L2	CO4

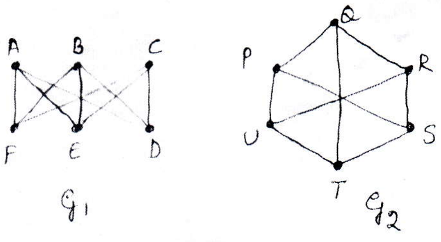
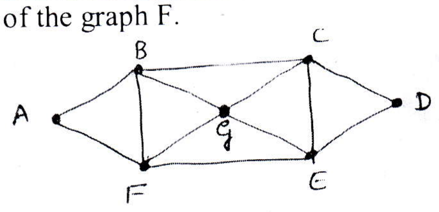
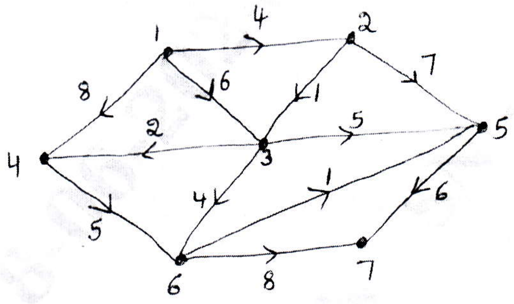
<table>
<tr><th colspan="6">Module – 4</th></tr>
<tr><td>Q.7</td><td>a.</td><td>Find the value of K such that the following distribution represents a finite probability distribution. Also find its mean and variance.
<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>P(x)</td><td>0</td><td>k</td><td>2k</td><td>2k</td><td>3k</td><td>k^2</td><td>$2k^2$</td><td>$7k^2+k$</td></tr></table></td><td>10</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>b.</td><td>When a coin is tossed 4 times, find the probability of getting, (i) exactly one head (ii) atmost 3 heads (iii) atleast two heads.</td><td>10</td><td>L3</td><td>CO2</td></tr>
<tr><th colspan="6">OR</th></tr>
<tr><td>Q.8</td><td>a.</td><td>Find K such that $f(x)=\begin{cases} kx^2, & -3\le x\le 3 \\ 0, & \text{elsewhere} \end{cases}$ is a probability density function.
Also find (i) $P(1\le x\le 2)$ (ii) $P(x>1)$</td><td>6</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>b.</td><td>In a certain town, the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that the shower will last for,
(i) 10 minutes or more
(ii) less than 10 minutes
(iii) between 10 and 12 minutes.</td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>c.</td><td>The marks of 1000 students in an exam follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be, (i) less than 65 (ii) more than 75 (iii) 65 to 75.
Given that $\phi(1)=0.3413$, where $z=\frac{x-\mu}{\sigma}$.</td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><th colspan="6">Module – 5</th></tr>
<tr><td>Q.9</td><td>a.</td><td>Define the following with an example for each :
(i) Simple graph
(ii) Regular graph
(iii) Spanning subgraph</td><td>6</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>Check whether the following graphs in Fig. Q9 (b) are isomorphic.


Fig. Q9 (b)</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Explain Konigsberg Bridge Problem.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><th colspan="6">OR</th></tr>
<tr><td>Q.10</td><td>a.</td><td>Define Euler circuit, Hamilton cycle and Planar graphs with an example for each.</td><td>6</td><td>L2</td><td>CO4</td></tr>
</table>

	b.	Give graph coloring of the graph F.  Fig. Q10 (b)	7	L2	CO4
	c.	Using the Dijkstra's algorithm. Obtain the shortest path from vertex 1 to each of the other vertices in the weighted, directed network shown below in Fig. Q10 (c)  Fig. Q10 (c)	7	L2	CO4

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